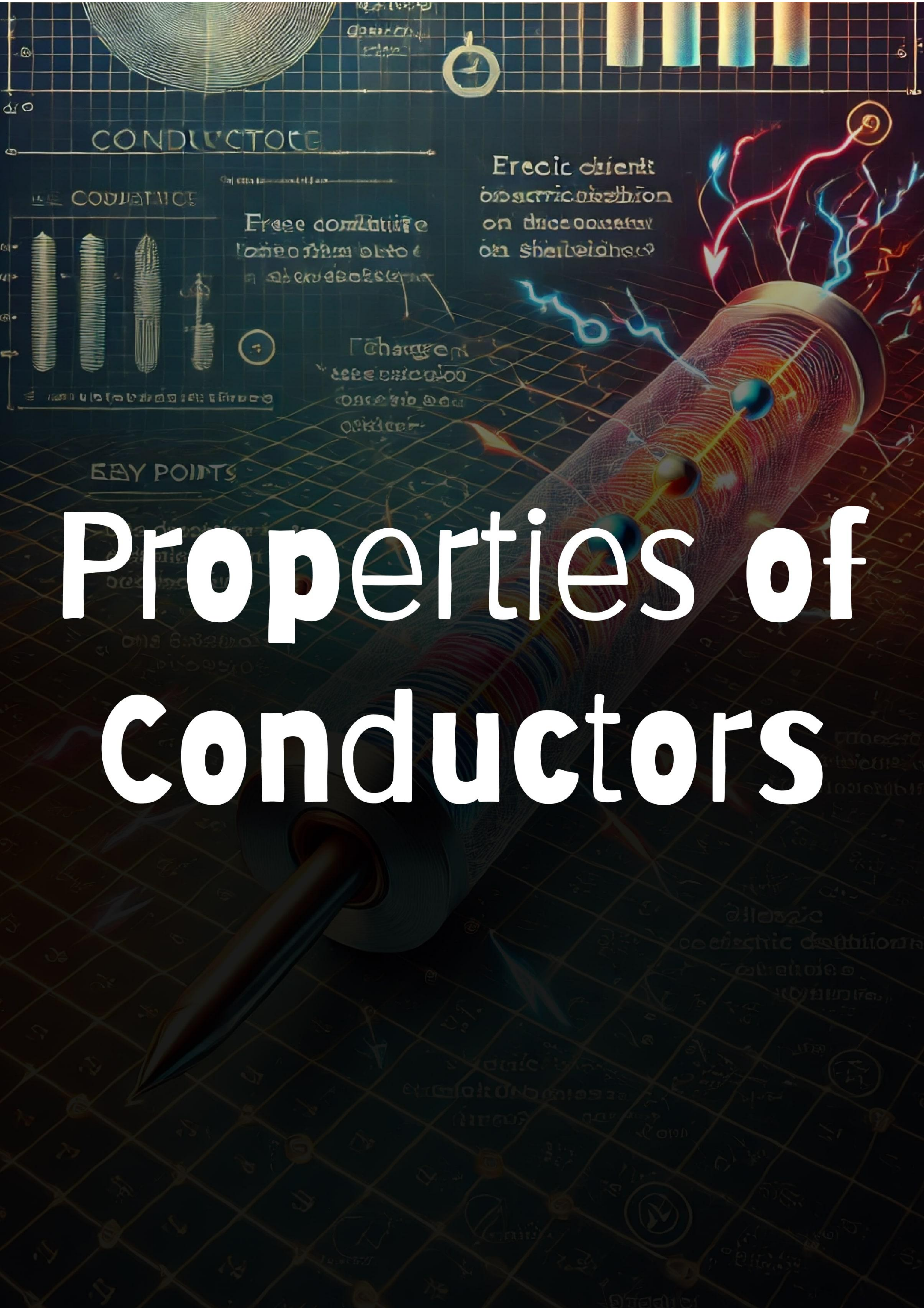


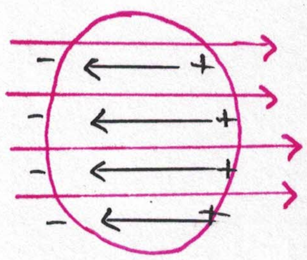
Properties of Conductors

Properties of Conductors

(Holds only when charges are at rest)

1.) The field in the matrix of a conductor is Zero.

Suppose if field passes through the conductor, then electrons will start moving till the field inside is zero.



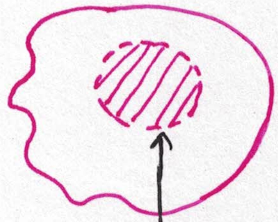
Electrons will get collected in left end. A new field is created inside which cancels the outside field.

Since in a conductor practically unlimited no. of free electrons are available, the electrons will not stop moving untill the field in the metal matrix becomes zero.

2.) All the charge given to a conductor resides on its surface.

Inside a metal conductor, $E = 0$

This can be shown by drawing a Gaussian surface in the metal matrix. Since field on this gaussian surface is zero, Therefore its associated flux is also zero, so it cannot enclose any net charge.



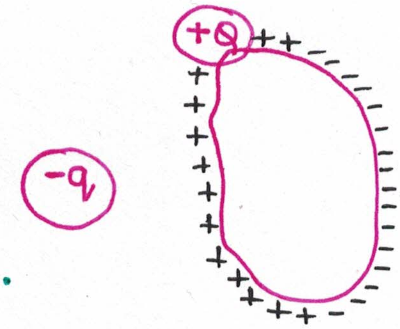
3.) INDUCTION & UNIQUENESS:

UNIQUENESS THEOREM

For a given configuration of the universe around a charged conductor, there exists one and only one configurt.ⁿ of the conductor for which the electric field in the matrix of the conductor is zero.

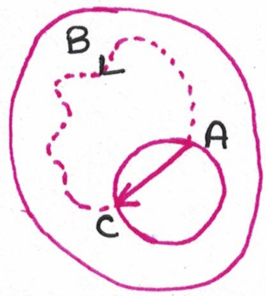
(i.e. the configuration is unique.)

When a conductor is placed in an external field, its charges rearrange themselves so as to create zero field in the conductor. This phenomenon is called **INDUCTION.**



4.) Field in the unoccupied cavity of a conductor is always zero.

If there exists a field line in the cavity, then work done is finite. But there exists another path through which work done is zero (through the conductor).



→ Since electric field in the matrix is zero, the entire conductor is at same potential. If we were to imagine a field line in the cavity, say A to C as shown in the fig. then work done in going from A to C directly along the field line will be non-zero whereas we can imagine another path ABC through the matrix of the conductor along which the work done is zero.

This contradicts the conservative nature of electrostatic field (i.e. work done should have been independent of path). Thus there cannot be any field lines in the cavity of the conductor.

5.) ELECTROSTATIC SHIELDING

Let Q_1 be the charge enclosed by the cavity.
Q_2 be the charge on the surface of the cavity.
Q_3 be the charge on the surface of the conductor.
Q_4 be the charge in the universe around the conductor.

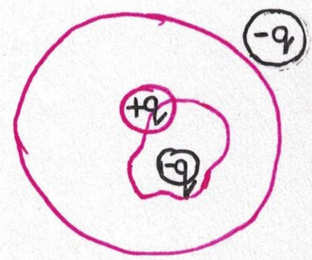
Further let E_1, E_2, E_3 & E_4 be the electric fields due to these charged configurations respectively.

A.) $E_1 + E_2 = 0$ in all the region outside the cavity.

B.) $E_3 + E_4 = 0$ in all the region enclosed by the conductor surface.

★ $Q_1 = -Q_2$ because if we assume a gaussian surface just outside the cavity, their field is zero, so flux is zero and hence charge is zero. So charge on surface of cavity is equal in magnitude to the charge enclosed inside the cavity but in opposite in sign.

Q) A neutral sphere has a cavity of arbitrary shape which encloses a charge $(-q)$. Comment on the charge configurat.? on the surface of the sphere.



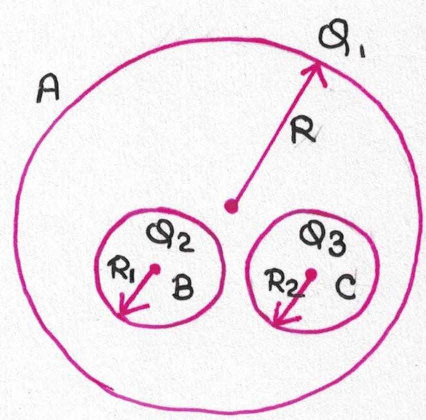
by charge conservation, outer surface of sphere will have $(-q)$ charge.

Contribution of the cavity charge & cavity surface charge to the region outside is zero. So we as well might forget these charges.

Now if this charge were to be uniformly distributed on the surface of the sphere, we will achieve zero field inside the sphere. Thus, this is the only configuration required (due to uniqueness theorem).

Que.) (a) What are the charges on the surfaces of cavities and outer surface of sphere?

(b) How does the field vary in the regions A, B and C? (as a function of r)



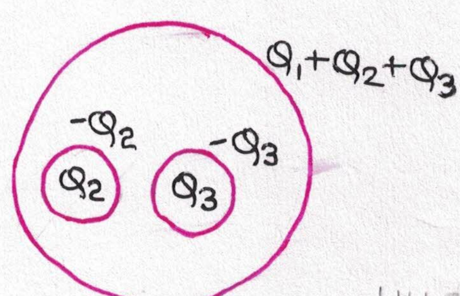
(a) Charge on surface of cavity $B = -Q_2$

charge on surface of cavity $C = -Q_3$

charge on surface of sphere $= Q_1 + Q_2 + Q_3$

(b) For region A,
(outside the sphere)

$$E_A = \frac{K(Q_1 + Q_2 + Q_3)}{r^2}$$



where r is measured from center of sphere
($\because$ sphere behaves as point charge for points outside sphere)

For region B, $E_B = \frac{KQ_2}{r^2}$

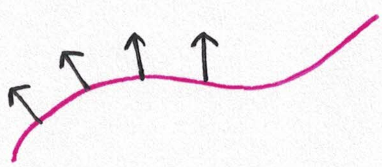
where r is measured from center of cavity B.

For region C, $E_C = \frac{KQ_3}{r^2}$

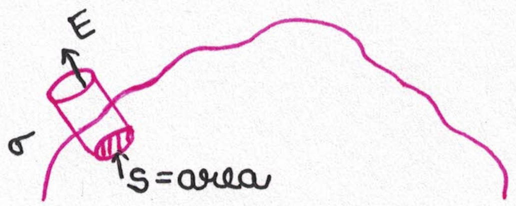
where r is measured from center of cavity C.

6.) Field lines on conducting surfaces are always $\perp$ to the surface.

If the field lines were not normal to the surface, then there will be a tangential component of the field along the metal surface leading to movement of electrons until the tangential component becomes zero.



7.) Field near the conducting surfaces:



σ = local charge density

Consider a metallic surface having local charge density σ as shown in the fig.

We make a pill-shaped gaussian surface near the surface of the conductor and apply Gauss law.

$$\phi = Es = \frac{\sigma s}{\epsilon_0}$$

or $$E = \frac{\sigma}{\epsilon_0}$$

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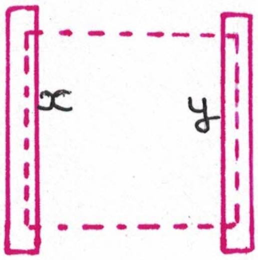
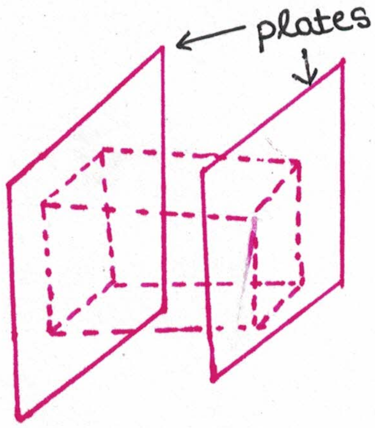
FACT: The facing surfaces of parallel infinite conducting sheets have equal and opposite charges.

Consider two parallel plates (conducting), having inner surface charges (x) & (y) as shown in fig. we construct a gaussian cuboid whose two faces pass through the matrix of the metal and other surfaces are parallel to the field such that it encloses the charge ($x+y$). The flux associated with the faces parallel to the field must be zero since $d\vec{A} \perp \vec{E}$ and the flux associated with the faces embedded in the metal is also zero as field in the metal is zero. Thus, net flux of this Gaussian cuboid is zero.

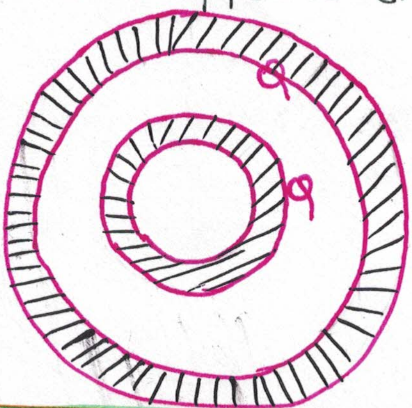
by applying Gauss law, $\frac{x+y}{\epsilon_0} = 0$

or $y = -x$

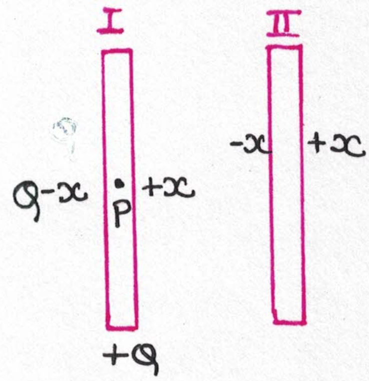




L The facing surfaces of spherical / long cylindrical shells have equal and opposite charges.



Que.) Two plates are separated by some distance as shown. Plate 1 is given a charge $(+Q)$ while plate 2 is maintained neutral. Find charge distribution on each surface.



Net field at point P as plate I is zero.

Field due to x on P = Left

Field due to $-x$ on P = Right

Field due to $+x$ on P = Left

Field due to $Q-x$ on P = Right

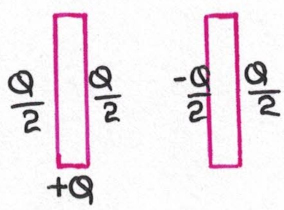
Since charge $x = |-x|$

$\therefore$ Their field cancel

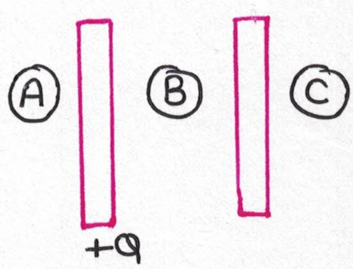
For infinite sheet, distance does not matter

$$\therefore Q - x = x$$

or $$x = \frac{Q}{2}$$



Que.) Find field in region (A), (B) & (C)



(A) $E_A = \dfrac{Q}{2A\epsilon_0}$ (Left)

(B) $E_B = \dfrac{Q}{2A\epsilon_0}$ (Right)

(C) $E_C = \dfrac{Q}{2A\epsilon_0}$ (Right)

For (A) $\sigma = \dfrac{Q}{2A}$ (due to $Q/2$)

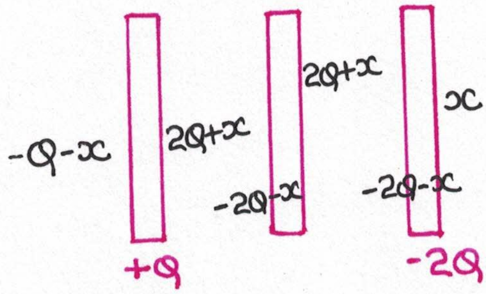
[A = area of plates]

$$\therefore E_{Q/2} = \frac{Q}{4A\epsilon_0}$$

$$\text{Total } E_P = \frac{Q}{4A\epsilon_0} + \frac{Q}{4A\epsilon_0}$$

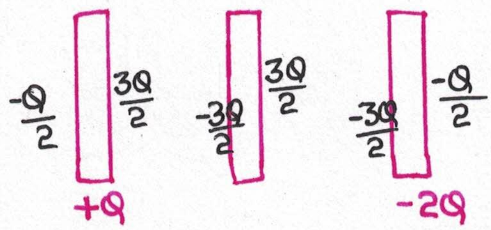
(due to the two faces of plate I)

Que.) Find charge appearing on right side of right most plate.



$$-Q - x = x$$

$$x = -\frac{Q}{2}$$



Que.) Find charge distribution on all surfaces.

$Q_1 = 0$

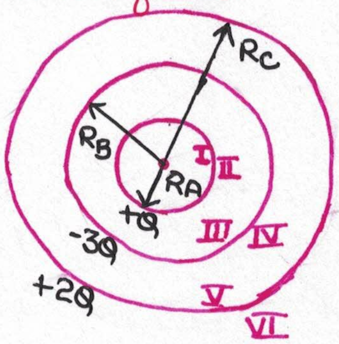
$Q_2 = +Q$

$Q_3 = -Q$

$Q_4 = -2Q$

$Q_5 = +2Q$

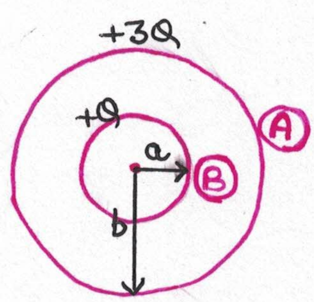
$Q_6 = 0$



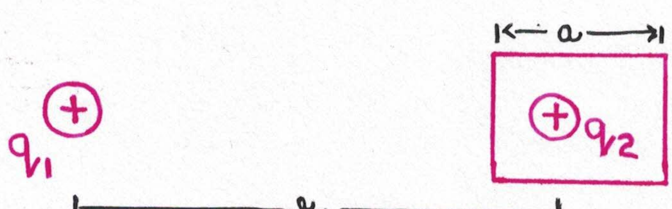
Que.) Find potential at shell A & shell B.

$$V_A = \frac{3KQ}{b} + \frac{KQ}{b} = \frac{4KQ}{b}$$

$$V_B = \frac{3KQ}{b} + \frac{KQ}{a}$$



Que.) Find force on q_2 due to free induced charge on cube surface.



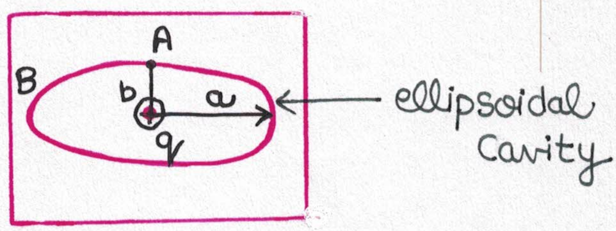
$$F_{q_2 q_1} = \frac{K q_1 q_2}{r^2} \quad (\longrightarrow)$$

$$\therefore (F_{q_2})_{\text{induced charge}} = \frac{K q_1 q_2}{r^2} \quad (\longleftarrow)$$

$\because$ Net field inside $= 0$

Net force on $q_2 = 0$

Que.)



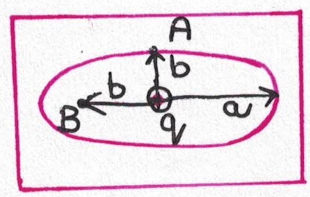
(a) $V_A > V_B$

(b) $V_B > V_A$

(c) $V_A = V_B$ ✓

(d) Data insufficient

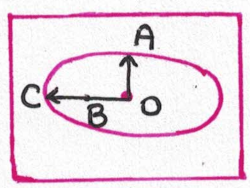
Que.)



(a) $V_A > V_B$

(b) $V_A < V_B$ ✓

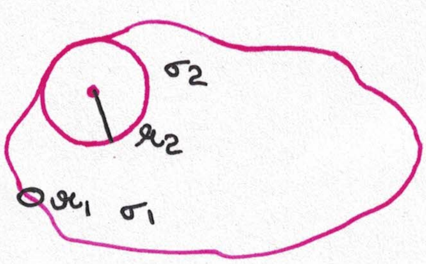
(c) $V_A = V_B$



$V_A = V_C$

$V_B > V_C$

$\because$ Potential decreases along field line.



$r_2 > r_1$

$\therefore \sigma_1 > \sigma_2$

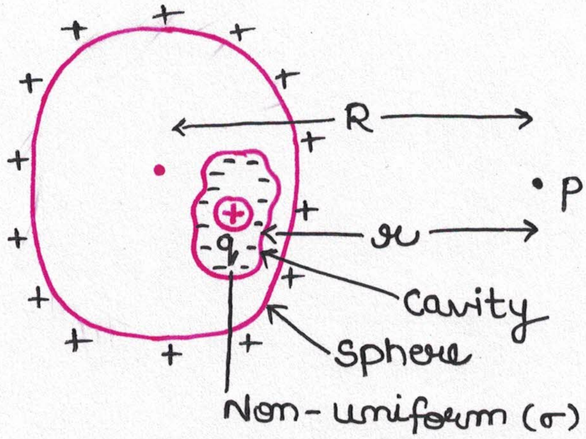
$\because \sigma r =$ Constant

(metal is equipotential)

For a metal, $V \propto \sigma r$

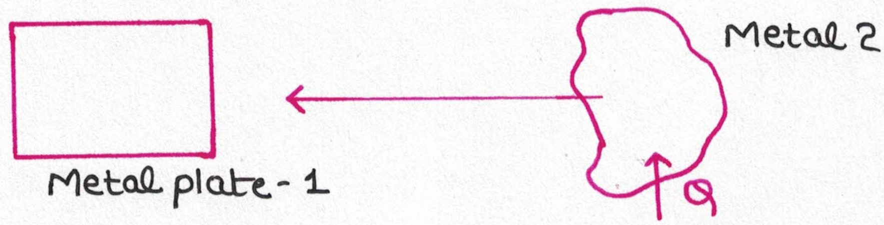
If r is less, then σ is large

$\therefore E$ is large.



$$(E_p)_{Net} = \frac{Kq}{R^2}$$

Que.) Let 'q' be charge flown 2 to 1 in first contact. Metal 2 is refilled to Q after each contact. If this process is repeated infinity, What is the amount of total charge that can be transferred to plate 1?



After 1^{st} contact,

K1 and K2 are different due to different shape of bodies.

q	$Q-q$
V_1	V_1
$V_1 = K_1 q$	$V_1 = K_2(Q-q)$

Finally,

q_∞	No flow	Q
V_∞		V_∞
$V_\infty = K_1 q_\infty$		$V_\infty = K_2(Q)$

$$\frac{\cancel{K_1} q}{\cancel{K_1} q_\infty} = \frac{\cancel{K_2}(Q-q)}{\cancel{K_2} Q}$$

$$\frac{Qq}{Q-q} = q_\infty$$

$$\boxed{q_\infty = \frac{Qq}{Q-q}}$$

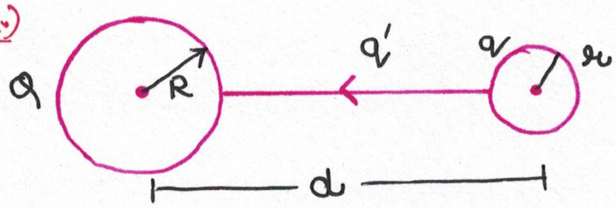
Que.) Calculate the charge flown during 2^{nd} contact.

$$\frac{\cancel{K_1} q}{\cancel{K_1}(q+q_2)} = \frac{\cancel{K_2}(Q-q)}{\cancel{K_2}(Q-q_2)}$$

$$\cancel{Qq} - \cancel{qq_2} = \cancel{Qq} - q^2 + Qq_2 - \cancel{qq_2}$$

$$q_2 = \frac{q^2}{Q}$$

Que.)



$R >> r$
$d >> R$

Let charge q' flow from r to R

$$\frac{K(Q+q')}{R} = \frac{K(q-q')}{r}$$

$$rQ + rq' = Rq - Rq'$$

$$q' = \frac{Rq - rQ}{(r+R)}$$

Now, $R + r \simeq R$

$$q' \simeq \frac{Rq}{R} - \frac{rQ}{R}$$

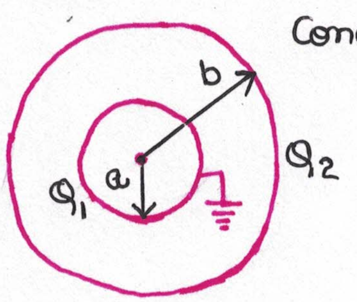
$$q' \simeq q - Q\left(\frac{r}{R}\right) \simeq q$$

$\therefore q' \simeq q$ ie all charge flows

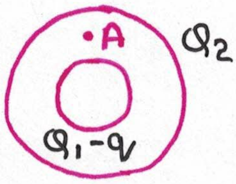
$\therefore (V_r) = 0$

ie $\lim_{r \to 0} q' = q$

Que.) Calculate the charge that will flow into the earth after the Key is closed.



Let q flow to earth



$$V_A = 0$$

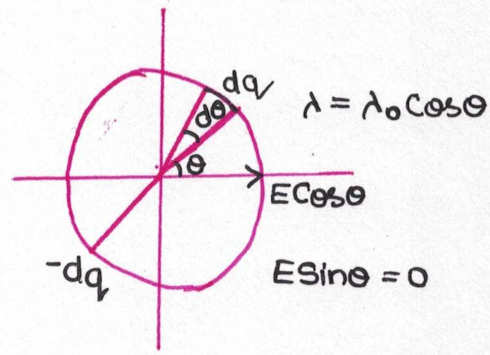
$$\frac{K(Q_1-q)}{a} = \frac{-KQ_2}{b}$$

$$Q_1 b - qb = -aQ_2$$

$$qb = aQ_2 + bQ_1$$

$$q = \frac{bQ_1 + aQ_2}{b}$$

Que.) Find dipole moment?

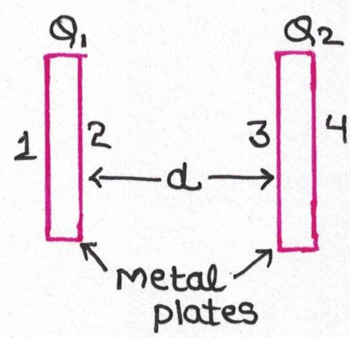


$$dp = \int (R\,d\theta)(\lambda_0 \cos\theta) \times 2R \times \cos\theta$$

$$= 2R^2 \lambda_0 \int \cos^2\theta \, d\theta$$

$$= 2R^2 \lambda_0 \left(\frac{\theta}{2} - 2\sin 2\theta\right)_{-\pi/2}^{\pi/2}$$

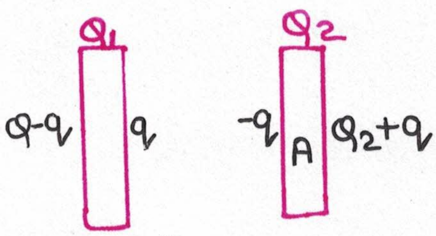
Que.)

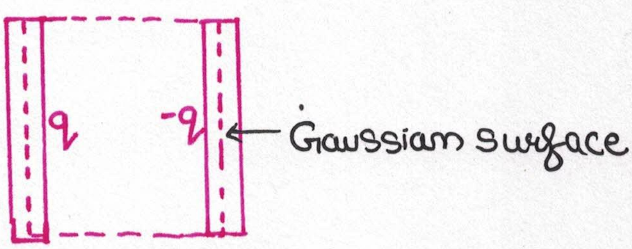


$$q_1 = \frac{Q_1+Q_2}{2} = q_4$$

$$q_2 = \frac{Q_1-Q_2}{2},$$

$$q_3 = \frac{Q_2-Q_1}{2}$$





$$E_A = 0$$

$$\frac{Q_1-q}{2A\epsilon_0} + \frac{q}{2A\epsilon_0} - \frac{q}{2A\epsilon_0} - \frac{Q_2+q}{2A\epsilon_0} = 0$$

$$Q_1 - q = Q_2 + q$$

$$q = \frac{Q_1-Q_2}{2}$$

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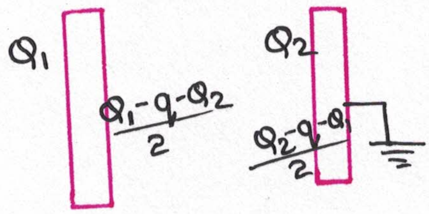
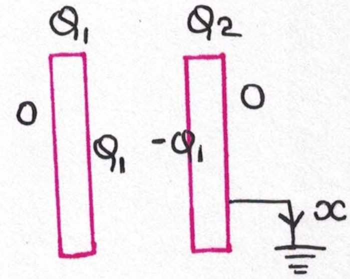
Que.) Calculate charge that will flow after Key is closed?

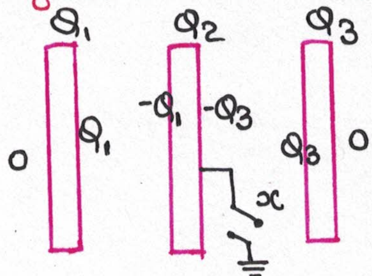


$$\underbrace{-Q_1}_{\text{left}} + \underbrace{x}_{\text{gone}} = Q_2$$

$$(x = Q_1 + Q_2)$$

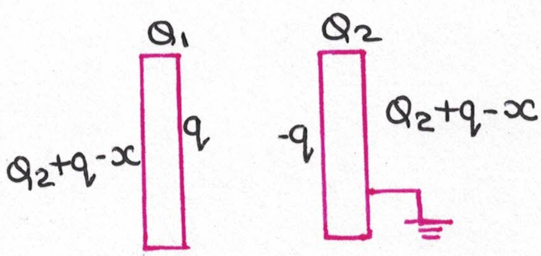


Que.) Charge flown into earth?



$$-Q_1 - Q_3 + x = Q_2$$

$$x = Q_1 + Q_2 + Q_3$$



$$Q_2 + q - x + q = Q_1$$

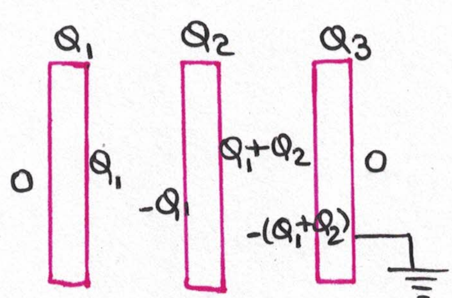
$$x = Q_2 - Q_1 + 2q$$

$$Q_2 + q - x = 0$$

$$q = x - Q_2$$

$$\therefore x = Q_2 - Q_1 + 2x - 2Q_2$$

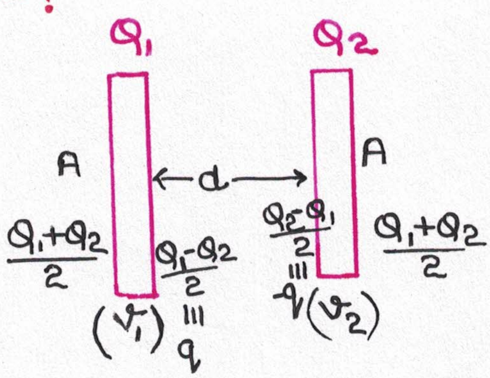
$$x = Q_1 + Q_2$$



$$-(Q_1 + Q_2) + x = Q_3$$

$$x = Q_1 + Q_2 + Q_3$$

Que.) $v_1 - v_2 = ?$



$\alpha \to$ small

$Q_1 > Q_2$

$v_1 - v_2 = ?$

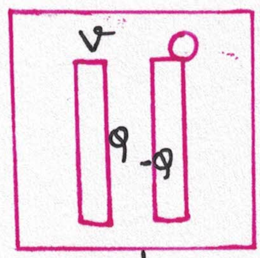
$$v_1 - v_2 = \frac{q}{A\epsilon_0} \cdot d = \frac{(Q_1 - Q_2)d}{2A\epsilon_0}$$

$$Q = \left(\frac{\epsilon_0 A}{d}\right) v$$

$$Q = CV$$

C = capacitance

Constant depends on geometry.

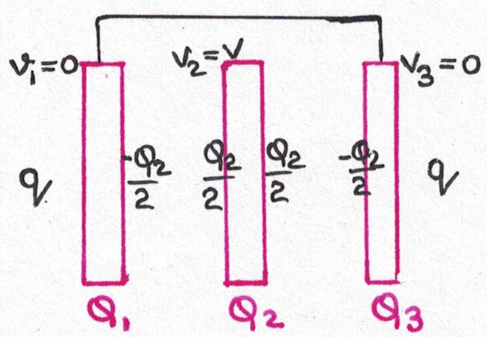


↓ System / Setup

⇓

Capacitor OR Conductor

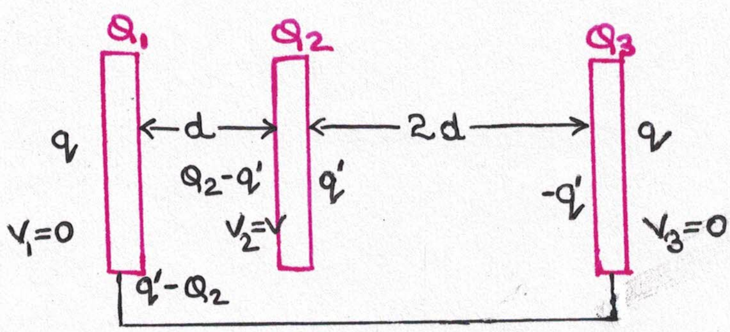
Que.)



$$2\left(q - \frac{Q_2}{2}\right) = Q_1 + Q_3$$

$$q = \frac{1}{2}\left[Q_1 + Q_2 + Q_3\right]$$

Que.)

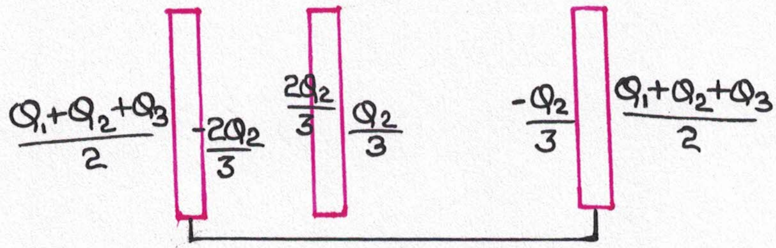


$$q' = \left(\frac{\epsilon_0 A}{2d}\right) v$$

$$Q_2 - q' = \left(\frac{\epsilon_0 A}{d}\right) v$$

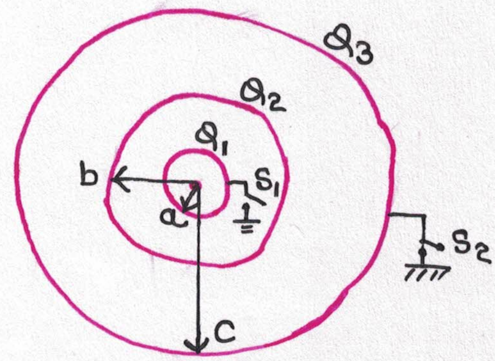
$$\frac{q'}{Q_2 - q'} = \frac{1}{2}$$

$$3q' = Q_2$$
$$q' = \frac{Q_2}{3}$$



$$2q - Q_2 = Q_1 + Q_3$$

Que.) In the shown situation, first the switch S_1 is closed & then switch S_2 is closed. Find charge flown through S_1 & S_2.



Let charge through $S_1 = x$
and charge through $S_2 = y$

$$\frac{K(Q_1-x)}{a} + \frac{KQ_2}{b} + \frac{K(Q_3-y)}{c} = 0 \quad -①$$

$$\frac{K(Q_3-y)}{c} + \frac{KQ_2}{c} + \frac{K(Q_1-x)}{c} = 0 \quad -②$$

from ②
$$Q_3 + Q_1 + Q_2 - (x+y) = 0$$
$$x = Q_1 + Q_2 + Q_3 - y$$

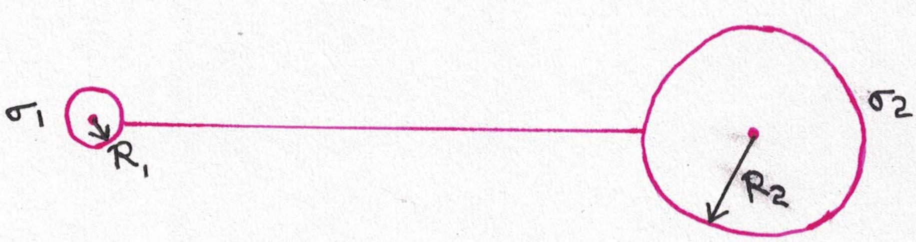
$$\frac{Q_1 - Q_1 - Q_2 - Q_3 + y}{a} + \frac{Q_2}{b} + \frac{Q_3 - y}{c} = 0$$

$$bcy - bcQ_2 - bcQ_3 + Q_2ac + Q_3ba - aby = 0$$

CORONA DISCHARGE

Consider two sphere having charge densities σ_1 & σ_2 and radii R_1 & R_2 separated by a large distance & connected by a conducting wire.

Let the spheres be conductors.



$$V_1 = \frac{K \times \sigma_1 \times 4\pi R_1^2}{R_1} = V_2 = \frac{K \times \sigma_2 \times 4\pi R_2^2}{R_2}$$

$$\sigma_1 R_1 = \sigma_2 R_2$$

$$\sigma_1 = \frac{\sigma_2 R_2}{R_1}$$

We see that if R_1 becomes very small then σ_1 becomes very large.

A large field may ionize the local air by pulling the electrons & thus the system will start getting discharged at the potential structure.

This phenomenon is called CORONA discharge.

Even though we have shown this for a system of two spheres, it is true for any conductor in general.

The highest chance of CORONA Discharge is at the point of lowest radius of curvature.